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Is the BELLE result for the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ a real difficulty for QCD ?

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Abstract

It is shown that difficulties in reconciling the values of the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ measured at BELLE and calculated within non-relativistic QCD (NRQCD) are caused not by some misinterpretation of the data or other exotic explanations, but by poor applicability of NRQCD to such processes.

We use general theory of hard exclusive processes in QCD together with more realistic models of charmonium wave functions, and show that the BELLE result can be naturally explained.

1. A surprisingly large rate for hard exclusive processes of the type $e^+e^- \to J/\psi + \eta_c$ observed at BELLE [1] still remains unexplained. In this experiment the process $e^+e^- \to J/\psi + X$ was studied. The cross-section of $e^+e^- \to J/\psi + \eta_c$ was then extracted from the number of events in the η_c peak in the mass spectrum of the system recoiling against the reconstructed J/ψ.

In a recent upgrade of the BELLE analysis with a data sample of $155\,\mathrm{fb}^{-1}$ [2],[3] the cross-section of the $e^+e^- \to J/\psi + \eta_c$ process has been found equal to $(25.6 \pm 2.8 \pm 3.4)\,\mathrm{fb}/Br(\eta_c > 2\,\mathrm{charged})$. BELLE also performed simultaneous fits to the production and helicity angle distributions. The measured angular and helicity distributions for $J/\psi + \eta_c$ have the general form $(1 + \alpha \cos^2\theta)$ and are consistent with the expectations for production of this final state via a single virtual photon, $\alpha_{prod} = \alpha_{hel} = +1$.

From the theoretical side, this cross section was calculated in [4] within the NRQCD approach, and much smaller value $\simeq 2.3\,\mathrm{fb}$ was obtained. This large discrepancy initiated further studies, both experimental and theoretical. Various explanations were proposed, e.g. that the two-photon production of $(J/\psi + J/\psi)$ can be significant and can imitate those of $J/\psi + \eta_c$ [5], [1] or even more exotic variants of the scalar gluonium production (for its mass happily coinciding with the charmonium energy levels) [6], etc.

The main purpose of this paper is to show that the origin of the discrepancy is due to a poor accuracy of NRQCD when applied to such processes. The main reason is that the charm quark is not sufficiently heavy and, as a result, the charmonium wave functions are not sufficiently narrow for a reasonable application of NRQCD to the description of charmonium production. And, as usual, hard exclusive processes are particularly sensitive to the widths of hadron wave functions. Below we describe the properties of the model wave functions of charmonium (which we consider as more realistic in comparison with the extreme δ-function-like wave functions of NRQCD), and show that the value of $\sigma(e^+e^- \to J/\psi + \eta_c)$ measured at BELLE can be naturally obtained.

2. The cross section of the process $e^+e^- \to \gamma^* \to J/\psi_\perp(p_1) + \eta_c(p_2)$ has the standard form:

$$\sigma\Big(e^+e^- \to \gamma^* \to J/\psi_\perp(p_1) + \eta_c(p_2)\Big) = \frac{\pi\alpha^2}{6}\left(\frac{|\vec{p}|}{E}\right)^3 Q_c^2\,|F_{VP}(s)|^2\,, \qquad (1)$$

[1] Later, this possibility was excluded by BELLE [2].

where $(|\vec{p}|/E)^3$ is the P-wave phase space factor and $Q_c = 2/3$ is the charm quark charge, while the form factor is defined as:

$$\langle \psi_\perp(p_1), \eta_c(p_2)|J_\mu(0)|0\rangle = \epsilon_{\mu\nu\rho\sigma} e_\perp^\nu p_1^\rho p_2^\sigma \, F_{VP}(s)\,. \tag{2}$$

Since there is one form factor only, the angular distribution is pure kinematical: $\sim (1+\cos^2\theta)$.

We will need only the asymptotic form of $F_{VP}(s)$ because $s = M^2_{\Upsilon(4S)} \simeq 112\,\mathrm{GeV}^2$ in the BELLE experiment.

General theory of hard exclusive processes in QCD has been developed in [7],[8] (see [9] for a review). In [7] it was obtained that at large $s = (p_1+p_2)^2$ the leading power term of the general two-hadron form factor has the following behaviour (up to logarithmic corrections):

$$\langle H_1(p_1, s_1, \lambda_1;\, H_2(p_2, s_2, \lambda_2)|J_\lambda|0\rangle \sim \left(\frac{1}{\sqrt{s}}\right)^{|\lambda_1+\lambda_2|+2n_{min}-3}. \tag{3}$$

Here: H_1 and H_2 are any two hadrons with momenta $p_{1,2}$, spins $s_{1,2}$ and helicities $\lambda_{1,2}$ in c.m.s., the current helicity (here, the projection of the photon spin onto direction $\vec{p}_1$) is: $\lambda = (\lambda_1 - \lambda_2) = 0,\, \pm 1$. n_{min} is the minimal number of point-like constituents (quarks or gluons) in these hadrons, $n_{min} = 2$ for mesons and $n_{min} = 3$ for baryons. It is seen that the asymptotic behaviour is independent of hadron spins, but depends essentially on their helicities.

For the process considered, $e^+e^- \to \gamma^* \to J/\psi_\perp(p_1) + \eta_c(p_2)$, the J/ψ-meson is transversely polarized, i.e. has helicities $|\lambda_1| = 1$ only. So, the matrix element in eq.(3) behaves as $\sim 1/s$. Since in eq. (2) $e_\perp \sim 1$, while $p_1 \sim p_2 \sim \sqrt{s}$, all that results in $F_{VP}(s) \sim 1/s^2$.

The leading term of $F_{VP}(s)$ is given by four similar diagrams, one of which is shown in Fig.1. Its explicit form will be given below in eq.(9,10), but we describe first the properties of the meson wave functions entering eq.(10).

3. The twist 2 and twist 3 light cone wave functions $V_i(x)$ and $P_i(y)$ of the $^3S_1$ and $^3S_0$ states of quarkonium made of the heavy $\overline{Q}Q$ quarks are defined in the standard way (see e.g. [9]), we neglect higher twist wave functions giving only power corrections in comparison with those written below. For simplicity, we also do not distinguish M_V from M_P and use $\overline{M}$ which can be taken as an appropriate average, for instance: $\overline{M} = (3M_V + M_P)/4$.

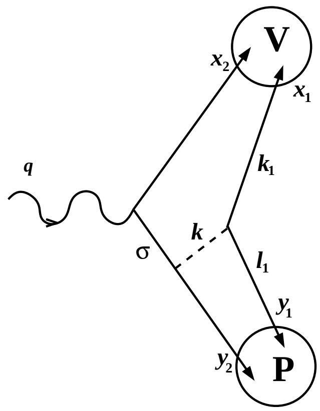

Figure 1: One of the four similar diagrams for the form factor $F_{J/\psi\,\eta_c}$.

For the V-meson with the helicity λ: [2]

$$\langle V_\lambda(p)|\overline{Q}_\beta(z)\,Q_\alpha(-z)|0\rangle_{\mu^*} = \frac{f_V\overline{M}}{4}\int_o^1 dx_1\, e^{i(pz)(x_1-x_2)}\Big\{\widehat{e}_\lambda\, V_\perp(x)+$$

$$+\,\widehat{p}\,\frac{(e_\lambda z)}{(pz)}\,\widetilde{V}(x) + f_v^t(\mu^*)(\sigma_{\mu\nu}e_\lambda^\mu\, p^\nu)\,V_T(x) + f_v^a(\mu^*)(\epsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_5\, e_\lambda^\nu\, p^\alpha z^\beta)\,V_A(x)\Big\}_{\alpha\beta}. \quad (4)$$

Taking $\lambda = 0$ in eq.(4) and using $\overline{M}e^\mu_{\lambda=0}\to p_\mu$ for large p, one obtains the standard definition of the leading twist wave function $V_L(x)$ of longitudinally polarized vector meson [9]:

$$\langle V_{\lambda=0}(p)|\overline{C}(z)\gamma_\mu C(-z)|0\rangle = f_V\int_o^1 dx_1\, e^{i(pz)(x_1-x_2)}V_L(x), \quad (5)$$

so that: $\widetilde{V}(x) = V_L(x) - V_\perp(x)$.

x_1 and $x_2 = (1-x_1)$ in eqs.(4,5) are fractions of the meson momentum $p^+ = (E+p_z)\equiv q_o$ carried by quarks at large p_z, i.e. $[+\,,\perp,\,-]$ components of the quark momentum are: $k_1 = [x_1q_o,\, 0_\perp,\, M_Q^2/x_1q_o]$ (and similarly for three other quarks), and we neglect the quark transverse momentum inside the heavy quarkonium in comparison with its mass.

[2] We use the conventions:

$$\sigma_{\mu\nu} = [\gamma_\mu,\gamma_\nu]/2,\quad \gamma_5 = (\gamma_5)_{Bj-Dr},\quad \widehat{p} = p_\mu\gamma^\mu = p_o\gamma_o - \vec{p}\,\vec{\gamma}.$$

For the P-meson:

$$\langle P(p)|\overline{Q}_\beta(z)\,Q_\alpha(-z)|0\rangle_{\mu^*} = i\frac{f_P\overline{M}}{4}\int_0^1 dx_1 e^{i(pz)(y_1-y_2)}\Big\{\frac{\widehat{p}\,\gamma_5}{\overline{M}}\,P_A(y)-$$

$$-\,f_p^p(\mu^*)\,\gamma_5\,P_P(y)+f_p^t(\mu^*)\,(\sigma_{\mu\nu}\,p^\mu\,z^\nu)\,P_T(y)\Big\}_{\alpha\beta}\,. \qquad (6)$$

The wave functions and operators entering eqs.(4-6) are defined at the soft scale μ^*, $\mu^*\ll q_B$, where $q_B=C_F\alpha_s(\mu\simeq q_B)M_Q^*/2$ is the Bohr momentum of the heavy quark in the quarkonium.

All wave functions in eq.(4-6) are symmetric under: $x_1\leftrightarrow x_2$, $y_1\leftrightarrow y_2$ and normalized: $\int_0^1 dx_1\,V_i(x)=1,\ i=\perp,\,L,\,T,\,A;\quad \int_o^1 dy_1\,P_i(y)=1,\ i=A,\,P,\,T$.

The values of dimensionless constants in the above formulae follow directly from the exact QCD equations of motion: $i\hat{D}Q=M_Q^*Q$ (see e.g. ch.9 and Appendix C in [9]; here M_Q^* is the "soft" mass of the heavy quark, e.g. the appropriately defined perturbative pole mass) [3] :

$$f_p^p(\mu^*)=\frac{\overline{M}}{2M_Q^*}\,,\quad f_v^t(\mu^*)=\Big(\frac{2M_Q^*}{\overline{M}}-\Delta_s\Big), \qquad (7)$$

$$f_v^a(\mu^*)=\frac{1}{2}\Big(1-f_v^t(\mu^*)\frac{2M_Q^*}{\overline{M}}\Big),\quad f_p^t(\mu^*)=\frac{1}{3}\Big(1-\frac{(2M_Q^*)^2}{\overline{M}^2}\Big).$$

However, the appropriate scale μ for the wave functions entering the hard form factor $F_{VP}(s)$ will be $\mu^2\simeq k^2$ rather than μ^{*2}, see Fig.1. For simplicity, we neglect in what follows the complicated but slow logarithmic evolution of (normalized) wave function forms, and will account only for the overall renormalization factors of the local tensor and pseudoscalar currents and for running of the quark mass. Unlike the larger soft mass M_Q^*, the mass $\overline{M}_Q$ entering below the perturbative logarithmic renormalization factors and couplings $f_{v,p}^i(k^2)$ will be the "hard" mass, e.g. $\overline{M}_Q=M_Q^{\overline{MS}}(\mu=M_Q^{\overline{MS}})$,

[3] Δ_s in eq.(7) originates from the matrix element: $\langle V_\lambda(p)|\overline{Q}(i\overrightarrow{D}_\mu-i\overleftarrow{D}_\mu)Q|0\rangle_{\mu^*}\equiv\Delta_s f_V\overline{M}^2e_\mu^\lambda$. It is $O(v^2)$ parametrically, and up to $O(v^4)$: $\Delta_s\simeq(4M_Q^{*2}-\overline{M}^2)/6\overline{M}M_Q^*$. For the c-quark pole mass $M_c^*=1.65\,\mathrm{GeV}$: $\Delta_s\simeq 0.05$, and will be neglected in what follows.

which is smaller because a part of the heavy quark self-energy in the interval $(1/\overline{M}_Q) < R < (1/\mu^*)$ is now excluded from M_Q^*. So, we use in eq.(10) (the renormalization factors Z_p, Z_t and Z_m^k are given below): [4]

$$f_p^p(k^2) = \frac{\overline{M}}{2\overline{M}_Q} Z_p, \quad f_v^t(k^2) = \frac{2\overline{M}_Q}{\overline{M}} Z_t, \quad f_v^a(k^2) = \frac{1}{2}\Big(1 - Z_t Z_m^k \frac{4\overline{M}_Q^2}{\overline{M}^2}\Big). \quad (8)$$

4. The leading contribution to the form factor $F_{VP}(s)$ is calculated in a standard way (see ch.9 in [9] where the calculation of the form factor $\gamma^* \to \rho_\perp + \pi$ is described in detail [5]), and the result for the leading term has the form:

$$|F_{VP}(s)| = \frac{32\pi}{9} \Big| \frac{f_V f_P \overline{M}}{q_o^4} \Big| I_o \,, \quad (9)$$

$$I_o = \int_0^1 dx_1 \int_0^1 dy_1 \alpha_s(k^2) \Big\{ \frac{Z_t Z_p V_T(x) P_P(y)}{d(x,y)\, s(x)} - \frac{\overline{M}_Q^2}{\overline{M}^2} \frac{Z_m^\sigma Z_t V_T(x) P_A(y)}{d(x,y)\, s(x)} +$$

$$+ \frac{1}{2} \frac{V_L(x)\, P_A(y)}{d(x,y)} + \frac{1}{2} \frac{(1-2y_1)}{s(y)} \frac{V_\perp(x)\, P_A(y)}{d(x,y)} + \quad (10)$$

$$+ \frac{1}{8} \Big(1 - Z_t Z_m^k \frac{4\overline{M}_Q^2}{\overline{M}^2}\Big) \frac{(1+y_1) V_A(x) P_A(y)}{d^2(x,y)} \Big\}.$$

Here: $s = 4E^2$, $q_o^2 = (|\vec{p}| + E)^2 \simeq (s - 2\overline{M}^2)$, $k = (k_1 + l_1)$ is the gluon momentum in Fig.1, $d(x,y)$ and $s(x)$ originate from the gluon and quark propagators, Z_t and Z_p are the renormalization factors of the local tensor and pseudoscalar currents:

$$d(x,y) = \frac{k^2}{q_o^2} = \Big(x_1 + \frac{\delta}{y_1}\Big)\Big(y_1 + \frac{\delta}{x_1}\Big), \quad \delta = \Big(Z_m^k \frac{\overline{M}_Q}{q_o}\Big)^2, \quad (11)$$

[4] The function $P_T(x)$ gives no contribution to the form factor $F_{VP}(s)$.

[5] The calculation for charmonium is even simpler because the three-particle $\overline{c}cg$-wave functions of twist 3 containing the additional valence gluon have couplings $f_3^i \sim f_V \Lambda_{QCD}^2/\overline{M}$, i.e. are suppressed by a factor $\sim (\Lambda_{QCD}/\overline{M})^2$ in comparison to $f_V\overline{M} \simeq f_P\overline{M}$ in eqs.(4-6), and can be neglected.

$$s(x) = \left(x_1 + \frac{Z_m^\sigma \overline{M}_Q^2}{y_1 y_2\, q_o^2} \right), \quad s(y) = \left(y_1 + \frac{Z_m^\sigma \overline{M}_Q^2}{x_1 x_2\, q_o^2} \right),$$

$$Z_p = \left[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)} \right]^{\frac{-3C_F}{b_o}}; Z_t = \left[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)} \right]^{\frac{C_F}{b_o}}; Z_m(\mu^2) = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\overline{M}_Q^2)} \right]^{\frac{3C_F}{b_o}}; \quad (12)$$

$$\overline{M}_Q(\mu^2) = Z_m(\mu^2)\, \overline{M}_Q\,, \qquad Z_m^k = Z_m(\mu^2 = k^2), \quad Z_m^\sigma = Z_m(\mu^2 = \sigma^2)\,, (13)$$

where $\overline{M}_Q(\mu^2)$ is the running $\overline{MS}$-mass, $C_F = 4/3$, $b_o = 25/3$.

5. For light quarks the asymptotic forms of wave functions entering eq.(4-6) look as follows (see ch.9 and Appendix B in [9]).
a) for the leading twist 2 wave functions:

$$P_A(x) = V_L(x) = V_T(x) = \phi_{asy}(x) = 6x_1x_2\,, \tag{14}$$

b) for the non-leading twist 3 wave functions:

$$P_P(x) = 1, \quad V_\perp(x) = \frac{3}{4}[1 + (x_1 - x_2)^2]\,, \tag{15}$$

$$V_A(x) = P_T(x) = 6x_1x_2\,.$$

For heavy quarkonium the light-front $1S$-Coulomb wave function can be taken as: [6]

$$\Psi(x,\, \vec{k}_\perp) \sim \left(\frac{\vec{k}_\perp^2 + (1 - 4x_1x_2)M_Q^{*\,2}}{4x_1x_2} + q_B^2 \right)^{-2},$$

$$\phi(x) \sim \int d^2\vec{k}_\perp\, \Psi(x,\, \vec{k}_\perp) \sim x_1x_2 \left\{ \frac{x_1x_2}{[1 - 4x_1x_2(1 - \bar{v}^2)]} \right\}. \tag{16}$$

Here: q_B is the Bohr momentum and $\bar{v} = q_B/M_Q^* \ll 1$ is the mean heavy quark velocity.

[6] It originates from the Schrödinger equal-time wave function $\Psi_{Sch}(r) \sim \exp(-q_B r) \to \Psi_{Sch}(\vec{k}) \sim (|\vec{k}|^2 + q_B^2)^{-2}$, supplemented with the proposed in [10] and commonly used substitution ansatz: $\vec{k}_\perp \to \vec{k}_\perp,\ k_z \to (x_1 - x_2)M_0/2,\ M_0^2 = (M_Q^{*\,2} + \vec{k}_\perp^2)/x_1x_2\,.$

For the leading twist charmonium wave functions we will use below a somewhat modified simple model form: [7]

$$\phi_o(x,v^2)=c_o(v^2)\,x_1x_2\left\{\frac{x_1x_2}{[1-4x_1x_2(1-v^2)]}\right\}^{1-v^2}, \tag{17}$$

$$\int_0^1 dx_1\phi_o(x,v^2)=1\,,$$

where v is now a parameter which has the meaning of the characteristic quark velocity in the bound state, while $c_o(v^2)$ is the normalization constant.

The wave function $\phi_o(x,v^2)$ in eq.(17) interpolates in the simplest way between two extreme cases: very heavy quark with $v\to 0$ in eq.(16), and very light quark with (formally) $v\to 1$ in eq.(14). In the non-relativistic case of very small $v\ll 1$ the wave function $\phi_o(x,v^2)$ is strongly peaked around the point $x_1=x_2=1/2$, so that $\phi_o(x,v^2\to 0)\to\delta(x_1-\frac{1}{2})$. And clearly, a decreasing quark mass leads to larger v and to wider $\phi_o(x,v^2)$. We take below $v^2=0.30$, as this value is commonly used in calculations for the $1S$ - charmonium.

The wave functions $\phi_o(x,v^2=0.30)$ from eq.(17) and $\phi_{asy}(x)=\phi_o(x,v^2=1)$ from eq.(14) are shown in Fig.2 (the wave function $\phi_o(x,v^2=0.08)$ corresponding to the scale of $\Upsilon(1S)$ is also shown for comparison [8]).

Guided by the above examples, we take for numerical calculations the following model wave functions of the $1S$-charmonium:

$$\phi_i(x,v^2)=c_i(v^2)\,\phi_i^{asy}(x)\left\{\frac{x_1x_2}{[1-4x_1x_2(1-v^2)]}\right\}^{1-v^2}. \tag{18}$$

With $v^2=0.30$ this looks as follows (compare to eqs.(14,15), all wave functions are normalized: $\int_0^1 dx\,\phi_i(x,v^2)=1$) :

[7] It is inspired by the fact that after taking into account small relativistic corrections the Coulomb wave function of the $1S$ two-particle bound state behaves at small r typically as: $\Psi(r)\sim r^{-\Delta}\exp(-q_Br)$. Here $\Delta=c_o\bar v^2+O(\bar v^4)$. The constant c_0 is not universal and e.g. for the ${}^1S_0$ -states of hydrogen and positronium: $\Delta_{hyd}=\bar v^2/2+O(\bar v^4)$, $\Delta_{pos}=4\bar v^2+O(\bar v^4)$. Therefore, this can be used only at really small $\bar v\ll 1$, such that $\Delta\ll 1$, and can't be taken literally for charmonium with $v^2\simeq 0.3$. So, we have taken the simplest model form of eq.(17) having in mind that it behaves qualitatively in a right way and will be really applied at $v^2\simeq 0.3$.

[8] It is seen from Fig.2 that even $\phi_o(x,v^2=0.08)$ is still far from the δ - function.

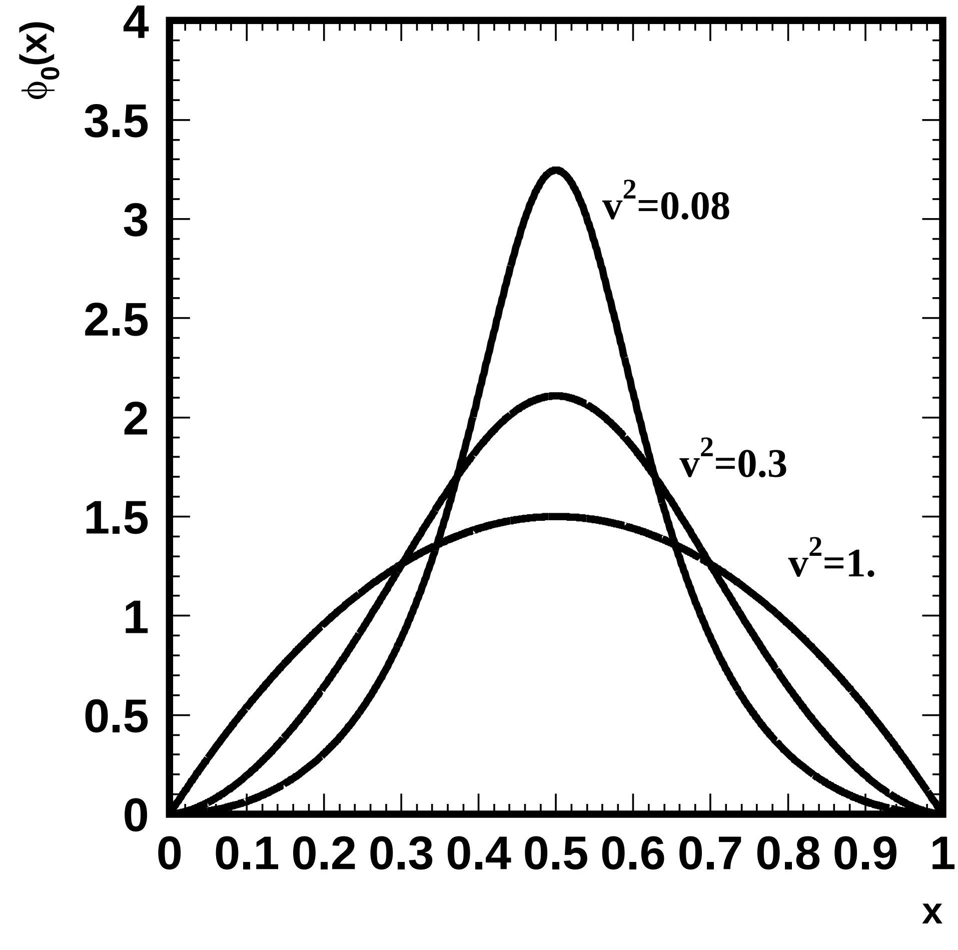

Figure 2: The shape of the wave function $\phi_o(x, v^2)$ for: $v^2 = 1$ – light quarks (asymptotic); $v^2 = 0.30$ – charmonium; $v^2 = 0.08$ – bottomonium.

a) for the leading twist 2 wave functions and $V_A(x)$:

$$V_T(x) = V_L(x) = P_A(x) = V_A(x) = 9.62\, x_1 x_2 \left(\frac{x_1 x_2}{1 - 2.8\, x_1 x_2} \right)^{0.70} . \qquad (19)$$

b) for the non-leading twist 3 wave functions:

$$P_P(x) = 1.97 \left(\frac{x_1 x_2}{1 - 2.8\, x_1 x_2} \right)^{0.70} ,$$

$$V_\perp(x) = 1.67\, [1 + (x_1 - x_2)^2] \left(\frac{x_1 x_2}{1 - 2.8\, x_1 x_2} \right)^{0.70} . \qquad (20)$$

5. For numerical calculations we use the following parameters. As for $\alpha_s(\mu^2)$, we take the simplest form: $(4\pi/b_o) \ln^{-1}(\mu^2/\Lambda^2)$ with $\Lambda = 200\,\mathrm{MeV}$. It gives, in particular: $\alpha_s(\mu^2 = M_\tau^2) \simeq 0.34$ at the scale of the τ-lepton mass, and this looks reasonable. For the $\overline{MS}$-mass of the c-quark we use the standard value: $\overline{M}_c = 1.2\,\mathrm{GeV}$, and we take $f_P \simeq f_V \simeq 400\,\mathrm{MeV}$ on average [9].

With all this and the wave functions given above in eqs.(19,20), one obtains from eqs.(1,9,10):

$$I_o \simeq 5.5\,, \quad F_{VP}(s = M^2_{\Upsilon(4S)}) \simeq 3.5 \cdot 10^{-3}\,\mathrm{GeV}^{-1},$$

$$\sigma\Big(e^+ e^- \to \gamma^* \to J/\psi_\perp + \eta_c\Big) \simeq 33\,\mathrm{fb}. \qquad (21)$$

This agrees with the BELLE result (assuming that $Br(\eta_c > 2\mathrm{charged})$ is not significantly less than unity). This fact can't be taken too literally, of course, due to dependence on the model form of used wave functions. Some check of the sensitivity of results to the wave function forms can be obtained by variation of the parameter v^2 in eq.(18). So, we repeated calculations with $v^2 = (0.30 \pm 0.05)$. Then the cross section changes only by $\pm 7\%$. Therefore, the results are not very sensitive to reasonable variations of v^2.

[9] The value of f_V is known from the leptonic decay of J/ψ: $\Gamma(J/\psi \to e^+e^-) = (16\pi\alpha^2/27)(|f_V|^2/M_\psi)$, $|f_V| \simeq 410\,\mathrm{MeV}$. The one-loop corrections ($\sim \alpha_s/\pi$) to the ratio $\Gamma(\eta_c \to 2\,\gamma)/\Gamma(J/\psi \to e^+e^-)$ indicate that f_P is slightly smaller than f_V.

Now, let us discuss some characteristic features of the above result for the integral I_o in eq.(10). The mean value of the renormalization factor Z_m^σ, see eqs.(12,13), is: $\langle Z_m^\sigma \rangle \simeq 0.74$. This means that in Fig.1 the typical value of the running c-quark mass in the quark propagator is: $\langle M_c(\mu^2=\sigma^2)\rangle \simeq 0.9\,\mathrm{GeV}$, while $\langle \sigma^2 \rangle \simeq 33\,\mathrm{GeV}^2$ which is ~ 1.5 times smaller than the typical value $\simeq q_o^2/2 \simeq 46\,\mathrm{GeV}^2$ for the narrow wave functions.

The value of $\langle k^2 \rangle$ can be inferred either from the mean value of $\langle Z_m^k \rangle \simeq 0.80$, or from the coupling $\alpha_s(k^2) : \langle \alpha_s(k^2)\rangle \simeq 0.263$. Both give the same result and show that the mean virtuality of the gluon in Fig.1 is: $\langle k^2 \rangle \simeq 12\,\mathrm{GeV}^2$. This is ~ 2 times smaller than a typical rough estimate: $\langle k^2 \rangle \simeq q_o^2/4 \simeq 23\,\mathrm{GeV}^2$ for narrow δ- like wave functions (and is not so far from the two-quark threshold $(2\overline{M}_c)^2 \simeq 5.8\,\mathrm{GeV}^2$). [10]

The smaller values of $\langle k^2 \rangle$ and $\langle \sigma^2 \rangle$ is the main reason why the standard NRQCD-calculations underestimate the cross section considerably. In other words, the charm quark is not very heavy and its wave functions are not much like the δ-functions, although they are of course significantly narrower than similar wave functions of really light quarks (see Fig.2).

It is also of interest to make a comparison to the value of the cross section obtained from eqs.(1,9,10) in the limit which, in essence, corresponds to the approximations of NRQCD, see [4]). For this, one has to replace in eq.(10): a) all wave functions, except for $V_A(x)$, by $V_i = P_i = \delta(x-1/2)$; b) to omit the term with $V_A(x)$; b) to replace all Z_i by 1. In this case:

$$k^2 \simeq [1 + (2\overline{M}_Q/q_o)^2\,]^2\,\frac{q_o^2}{4} \simeq 26\,\mathrm{GeV}^2, \quad \alpha_s(k^2) \simeq 0.23\,. \tag{22}$$

Substituting all this into eqs.(1,9,10), one obtains:

$$I_o \simeq 1.6\,, \quad F_{VP} \simeq 1.1 \cdot 10^{-3}\,\mathrm{GeV}^{-1}\,, \quad \sigma \simeq 3\,\mathrm{fb}, \tag{23}$$

which is essentially smaller than in eq.(21). [11]

On the whole, as was argued above, the difficulties with explaining the BELLE result for $\sigma(e^+e^- \to J/\psi + \eta_c)$ are not really the difficulties of QCD,

[10] It is worth noting also that while $f_v^a(\mu^*)$ in eq.(7) is very small and negative (e.g., using for the c-quark pole mass the value $M_c^* = 1.65\,\mathrm{GeV}$ obtained in [11] from the D-meson semileptonic width, one obtains: $f_v^a(\mu^*) \simeq -0.08$), the coupling $f_v^a(k^2)$ from eq.(8) becomes positive and much larger, $\langle f_v^a(k^2)\rangle \simeq +0.26$, so that the contribution of the term with the wave function $V_A(x)$ in eq.(10) is $\simeq 15\%$ of the whole integral I_o.

[11] In the opposite limit when all $1S$ - charmonium wave functions are taken as the asymptotic ones, see eqs.(14,15), the cross section will be $\sigma \simeq 70\,\mathrm{fb}$.

but are rather due to a poor approximation of the real dynamics of c-quarks by NRQCD. Within the approach described in this paper (which we consider as more realistic), the BELLE results look rather natural.

We hope that subsequent experimental and theoretical efforts in this field will help to elucidate dynamics and properties of various charmonium states. In particular, the use of the wave functions $V_i(x)$ given in eqs.(19,20) instead of $\delta(x-0.5)$ from NRQCD, will enhance the calculated cross sections of inclusive direct production of J/ψ.

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References

[1] K. Abe et al. (Belle Collab.), "Observation of Double-$\bar{c}c$ Production in e^+e^- Annihilation at $\sqrt{s}\sim 10.6\,\mathrm{GeV}$", Phys. Rev. Lett. **89** (2002) 142001; hep-ex/0205104

[2] K. Abe et al. (Belle Collab.), "Study of double charmonium production in e^+e^--annihilation at $\sqrt{s}\sim 10.6\,\mathrm{GeV}$", Phys. Rev. **D70** (2004) 071102; hep-ex/0407009

[3] P. Pakhlov, "Further study of double charmonium prodution in e^+e^--annihilation at Belle", hep-ex/0412041.

[4] E. Braaten, J. Lee, "Exclusive Double Charmonium Production from e^+e^-- Annihilation into a Virtual Photon", Phys. Rev. **D67** (2003) 054007; hep-ph/0211085

[5] G.T. Bodwin, J. Lee, E. Braaten, "e^+e^--Annihilation into $J/\psi+J/\psi$" , Phys. Rev. Lett. **90** (2003) 162001, hep-ph/0212181; "Exclusive Double-Charmonium Production from e^+e^--Annihilation into Two Virtual Photons", Phys. Rev. **D67** (2003) 054023, hep-ph/0212352

[6] S.J. Brodsky, A.S. Goldhaber, J. Lee, "Hunting for glueballs in electron-positron annihilation", Phys. Rev. Lett. **91** (2003) 112001, hep-ph/0305269

[7] V.L. Chernyak, A.R. Zhitnitsky, "Asymptotic behaviour of hadron form factors in the quark model", JETP Lett. **25** (1977) 510;
V.L. Chernyak, V.G. Serbo, A.R. Zhitnitsky, "Asymptotic behaviour of hadron form factors in QCD", JETP Lett. **26** (1977) 594.

[8] V.L. Chernyak, A.R. Zhitnitsky, "Asymptotic behaviour of hadron form factors in QCD", Sov. J. Nucl. Phys. **31** (1980) 544;
V.L. Chernyak, V.G. Serbo, A.R. Zhitnitsky, "Calculation of the pion form factor asymptotic behaviour in the QCD perturbation theory", Sov. J. Nucl. Phys. **31** (1980) 552.

[9] V.L. Chernyak, A.R. Zhitnitsky, "Asymptotic behaviour of exclusive processes in QCD", Phys. Rep. **112** (1984) 173.

[10] M.V. Terent'ev, "About structure of meson wave functions as bound states of relativistic quarks", Sov. J. Nucl. Phys., **24** (1976) 106.

[11] V.L. Chernyak, "Calculation of the D- and B-meson lifetimes", Nucl. Phys. **B457** (1995) 96, hep-ph/9407353